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Identification and an Inverse Filtering Problem

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Abstract: This paper is devoted to the problem of compensating for errors of nonlinear dynamical systems (DSs) between observation sessions. Such a problem is connected with the coordination of solutions of the nonlinear differential equations for the DSs and the linearized equations for their errors. The stochastic nature of such equations leads to the necessity of controlling the process of compensation for error estimates to stabilize the DS parameters with respect to the reference phase path. The proposed solution to the above problem is based on the application of the method of inverse problems of dynamics to the synthesis of a loop intended to compensate for the estimates being formed by a filter. When implementing the method mentioned, the need arises for identification of the coefficients of estimate damping. Procedures for such identification are discussed. The results of experimental studies are given.

Keywords: nonlinear system, Kalman filter, error damping, inverse problem of dynamics, identification of coefficients.

1. INTRODUCTION

At present, the application of an extended Kalman filter (EKF) to the estimation of errors of nonlinear dynamical systems (DSs) is considered to be justified, see Maybeck (1982). The technology of estimation relies on a mathematical description of the functioning of both the reference (unperturbed) DS and an actual (perturbed) DS. The ideal vector $Y(t)$ and the actual vector $Y_a(t)$ of state parameters are made to correspond to such DSs; these vectors are described by the following differential equations:

for the ideal DS: $dY(t)/dt = \dot{Y}(t) = F[Y(t)]$; (1)

for an actual DS: $\dot{Y}_a(t) = F[Y_a(t)] + G(t)\xi(t)$, (2)

where $\xi(t) = [\xi_1(t) \dots \xi_r(t)]^T$ is the vector of disturbances that affect the DS, which is characterized by the covariance matrix $E[\xi(t)\xi^{\mathrm{T}}(t-\tau)] = Q(t)\delta(t-\tau)$; $\delta(t-\tau)$ is the delta-function; $E[\dots]$ is the operator of mathematical expectation; $G(t)$ is the matrix of disturbance intensities.

Parameters of the ideal DS and the actual DS are related by the following error equation:

$$dx(t)/dt = \dot{x}(t) = A(t)x(t) + G(t)\xi(t), \tag{3}$$

where $x(t) = \Delta Y(t) = Y_a(t) - Y(t)$ is the vector of DS errors; $A(t) = \partial F[Y(t)]/\partial Y\big|_{Y(t) = Y_a(t)}$ is the matrix of coefficients that characterize the dynamics of variation of DS errors.

The estimates $\hat{x}(t)$ of DS errors are obtainable through the use of the EKF by processing the following observations:

$$z(t) = h[Y_a(t)] - h[Y(t)]_{\mathrm{SEI}}, \tag{4}$$

where $h[Y(t)]_{\mathrm{SEI}}$ is an observed value formed by the sensor of information that is external (SEI) with respect to the DS, and the above observed value has the model $h[Y(t)]_{\mathrm{SEI}} = h[Y(t)] + \vartheta(t)$;

$\vartheta(t)$ is the vector of perturbations in a measuring channel, which has the covariance matrix $M[\vartheta(t)\vartheta^T(t-\tau)] = R(t)\delta(t-\tau)$.

In the EKF, the interrelation of observations (4) and DS errors is taken into account via the following mathematical model:

$$z(t) = H(t)x(t) + \vartheta(t), \tag{5}$$

where $H(t) = \partial h[Y(t)]/\partial Y\big|_{Y(t) = Y_a(t)}$ is the matrix for the relation of observed parameters and the vector of DS errors.

The DS under observation, which has the EKF in an error estimation loop can be represented by a diagram shown in Figure 1, where U is the vector of control actions;

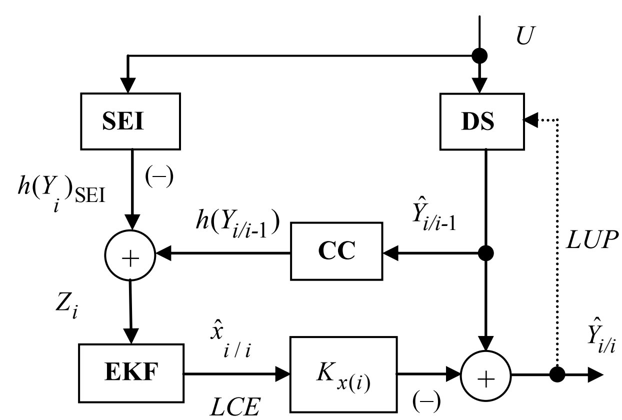

Fig. 1. Block diagram of an observable DS with the EKF

CC is a coordinate converter; $\wedge$ is the symbol for estimate; $\hat{Y}_{i/i-1}$; $\hat{x}_{i/i-1}$ are the predicted values of estimates both of the vector of parameters and of the DS errors at the instant $t = t_i$

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of time after Z_{i-1} observations have been processed; $\hat{Y}_{i/i}$; $\hat{x}_{i/i}$ are the updated values of estimates after Z_i observations have been processed; K_x are the coefficients of estimate damping; *LCE* is the loop for compensating of estimates; *LUP* is the loop for the updating of parameters. The above-mentioned coefficients set the level of confidence in the estimates being formed when the stochastic nature of the observations *Z*(*t*) and the predicted dynamics of error variation are taken into account. It should be noted that as a SEI, use can be made of the mathematical model of the reference DS.

For estimates, the discrete representation of Eqs. (2) and (3) has the following form:

$$\hat{Y}_{i/i-1} = \widetilde{F}(\hat{Y}_{i-1/i-1}, t_i)\,; \tag{6}$$

$$\hat{x}_{i/i-1} = \Phi_i \hat{x}_{i-1/i-1}, \tag{7}$$

Φ_i is the transition matrix for the vector of errors, and this matrix is determined from the solution of the differential equation $\dfrac{d\Phi(t,t_{i-1})}{dt} = A(t)\Phi(t,t_{i-1})$ for $\Phi(t_{i-1},t_{i-1}) = I$; I is an identity matrix of the appropriate dimension.

Typical equations that describe the functioning of a DS with loops intended for optimal estimation of errors in discrete time are the following ones:

- diagram with a loop for compensating for estimates (see Fig.1, without a dashed line):

$$\hat{x}_{i/i} = \hat{x}_{i/i-1} + K_i(z_i - H_i\hat{x}_{i/i-1})\,;\ \hat{Y}_{i/i} = \hat{Y}_{i/i-1} - \hat{x}_{i/i}\,; \tag{8}$$

- diagram with a loop for the updating of parameters (see Fig.1, with a dashed line):

$$\hat{x}_{i/i} = K_i z_i\,;\ \hat{Y}_{i/i} = \hat{Y}_{i/i-1} - \hat{x}_{i/i}\,; \hat{x}_{i/i} := 0\,, \tag{9}$$

where K_i is the EKF amplification factor.

The diagram with a loop for compensating for estimates (8) corresponds to the indicator mode of integration of the DS and SEI. In this mode, the necessary conditions for EKF functioning, which are associated with the linearity of the errors being estimated are not fulfilled. When the loop for the updating of DS parameters is implemented by the algorithm (9), complete compensation for the estimates of errors and their subsequent reduction to zero is carried out. In this case, however, the module for prediction of errors (7), which maintains EKF smoothing properties is eliminated from the EKF structure. Because of this, the necessity arises of forming the procedures for adaptive control of estimates between the points in time at which the observations are formed. Such control can be realized using the coefficients K_x of estimate damping (see Fig.1). The above coefficients must take account of the stochastic nature of estimates, and also they must maintain the values of DS errors with respect to the reference phase path, which are acceptable for a linearized EKF.

The purpose of this paper is concerned with the synthesis of coefficients of estimate damping, which take account of the conditions for integration of the DS and SEI. The accomplishment of the purpose formulated relies on the technology of inverse problems of dynamics, see Krayl'uk et al. (2017).

2. AN INVERSE FILTERING PROBLEM AND IDENTIFICATION OF THE COEFFICIENTS OF ESTIMATE DAMPING

Stated classically, the direct and inverse problems of dynamics (IPD) can be interpreted in the following way:
- the direct problem is: for given initial conditions and given input actions, determine the path of system motion in the phase space. For system (3), such a path is described by a matrix exponential;
- the inverse problem is: determine input actions that ensure system motion along the given path in the phase space. If the phase path is given in the form of a matrix exponential, the problem consists in determining the coefficients *A*(*t*) in Eq. (3), which provide for the motion along this exponential. For system (3), which has fixed parameters *A*(*t*), it is essential to determine additional coefficients K_x, which minimize the deviation of the reckoned phase path $\hat{Y}(t_i)$ from the reference path $Y(t_i)$. To such an interpretation there corresponds the following modification of Eq. (6), i.e.,

$$\hat{Y}_{i/i-1} = \widetilde{F}(\hat{Y}_{i-1/i-2}, \Delta t_i) + u_i\,, \tag{10}$$

where $\Delta t_i = t_i - t_{i-1}$; $u_i = -K_{x(i)}\hat{x}_{i/i-1}$. (11)

The synthesis of control (11) can be considered as a solution of the inverse filtering problem (IFP). It should be noted that unlike a traditional IPD, information about the reference phase path for the IFP is inaccessible. Because of this, to solve such a problem, one needs to use alternative approaches that do not require knowledge of the reference phase path in an explicit form.

For forming the matrix of the compensation coefficients $K_{x(i)}$, a technology for the exponential reduction of errors in a nonlinear observable system can be used. In this case, $K_{x(i)} = \Lambda_i$, where $\Lambda_i = diag(\lambda_{1(i)} ... \lambda_{n(i)})$ is the diagonal matrix of error damping coefficients $(\lambda_{j(i)} > 0)$, and the combination of Eqs. (7), (10), and (11) assumes the following recursion form:

$$\hat{Y}_{i/i-1} = \widetilde{F}(\hat{Y}_{i-1/i-2}, \Delta t_i) - \Lambda_i \hat{x}_{i/i-1}; \tag{12}$$

$$\hat{x}_{i+1/i} = \Phi_i(1-\Lambda_i)\hat{x}_{i/i-1} = \widetilde{\Phi}_i \hat{x}_{i/i-1}, \tag{13}$$

where $\hat{x}_{i/i-1}; \hat{x}_{i+1/i}$ are the predicted estimates of the vector of DS errors at the step of integration.

In this paper, a two-level procedure for optimization of the damping loop is proposed. At the first level, robust – with respect to outlying observations – estimates of DS errors are formed, and at the second level, coefficients of error damping are formed. The basis of an algorithm for robust estimation is the generalized parameter $\beta_j = \nu_j / \alpha_j$, where $\nu_j = Z_j - \hat{Z}_j$ is the residual between the actual value Z_j and the predicted value $\hat{Z}_j = H_j m_j$ of observations; m_j, $\hat{x}_j$ are the estimates of the vector of DS errors after processing the *j*-th component and the whole vector of observations; H_j is the row vector of coefficients of coupling between the observation Z_j and the vector of errors; α_j is a scale parameter. A robust modification of the EKF (RKF) results from the solution of the following problem:

$$\hat{x}_i = \underset{x_i}{\operatorname{argmin}} \sum_{i=i_0}^{i_f} \rho(\beta_i) \text{ under the constraint } x_i = \Phi_i x_{i-1}, \quad (14)$$

where $\rho(\beta_j) = -\ln f(\beta_j)$ is a likelihood function; $f(\beta_j)$ is a probability density function (PDF).

The solution of the problem (14), in expanded form, is given in Chernodarov (2009). For the Gaussian PDF, the solution is an EKF. The novelty of the paper under consideration lies in the algorithm for optimal damping of DS errors, which is included in the structure of a robust estimation filter.

In the realization of Eqs. (12) and (13), the problem of determining the estimates $\hat{\bar{\Lambda}}_i = [\hat{\lambda}_{1(i)} \ldots \hat{\lambda}_{n(i)}]^{\mathrm{T}}$ of the vector of coefficients, which are optimal ones according to the appropriate criterion still remains topical. We propose that the optimality criterion should be constructed on a basis of the residuals η_i between the "a priori" (predicted) estimates $\hat{x}_{i/i-1}$ and the "a posteriori" (after processing the observations) estimates $\hat{x}_{i/i}$ of the vector of DS errors, i.e.,

$$\hat{\bar{\Lambda}}_i = \arg \min_{\bar{\Lambda}_i} 0.5 \sum_{i=i_0}^{i_f} \eta_i^{\mathrm{T}} P_{i/i}^{-1} \eta_i , \quad (15)$$

provided that $\bar{\Lambda}_i = \bar{\Lambda}_{i-1}$,

where $\nu_i = \Phi_i x_{i-1} - x_{i/i}$; $\eta_i = \nu_i - \Phi_i \Lambda_i x_{i-1}$;

$P_{i/i}$ is the covariance matrix of estimation errors, which was formed at the *i*-th instant of time after *i* observations have been processed.

The solution of the problem (15) was obtained using Pontrjagin′s method, see Sage and Melse (1976). The application of this method is based on the formation of the Hamiltonian.

$$\mathcal{H} = 0.5\{\|\eta_i\|^2_{P^{-1}_{i/i}} + 0.5\|\omega_i\|^2_{N^{-1}_i}\} + \mu_i^T (\bar{\Lambda}_{i-1} + \omega_i) \quad (16)$$

and on the solution of the canonical equations

$$\frac{\partial \mathcal{H}}{\partial \mu_i^T} = \hat{x}_i ; \quad \frac{\partial \mathcal{H}}{\partial x_{i-1}} = \hat{\mu}_{i-1}^T ; \quad \frac{\partial \mathcal{H}}{\partial \omega_i} = 0 \quad (17)$$

with the boundary conditions $\hat{\mu}_{i_f} = 0$; $\hat{x}_{i_0} = 0$, (18)

where $\|\eta_i\|^2_{P^{-1}_{i/i}} = \eta_i^T P_{i/i}^{-1} \eta_i$; μ_i is the vector of Lagrange multipliers; $\omega_i = \omega(t_i) = [\omega_1(t_i) \ldots \omega_k(t_i)]^{\mathrm{T}}$ is the vector of disturbances that affect the compensation coefficients, which is characterized by the covariance matrix $E[\omega(t)\omega^{\mathrm{T}}(t-\tau)] = N(t)\delta(t-\tau)$.

Upon computation of partial derivatives and on execution of algebraic transformations, we can obtain the following expanded form of the canonical equations:

$$\hat{\Lambda}_{i/i-1} = a = \hat{\Lambda}_{i-1/i-1} - N_{i-1}\hat{\mu}_{i-1} ; \quad (19)$$

$$\hat{\mu}_i = b = \hat{\mu}_{i-1} + \tilde{x}_{i-1}^T \Phi_i^T P_{i/i}^{-1} \eta_i ; \quad (20)$$

$$\hat{\omega}_i = -Q_{i-1}\hat{\mu}_{i-1} , \quad (21)$$

where $\tilde{x}_{i-1} = diag\{x_{1(i-1)}, \ldots, x_{n(i-1)}\}$.

For finding the estimate $\hat{x}_{i/i}$ of the vector of errors at the *i*-th instant of time from *i* observations, we need only solve a two-point boundary problem (TPBP), which is determined by Eqs. (19) - (21) and boundary conditions (18). The solution of the above TPBP can be found by the method of invariant imbedding, see Sage and Melse (1976). According to this method, when the TPBP is included in a more general problem with the boundary conditions

$$\hat{\mu}_{i_f} = c ; \quad \hat{x}_{i_f}[c, i_f] := x_{i_f} - S_{i_f} c , \quad (22)$$

where $\hat{x}_{i_f}$ is the solution of the TPBP for c = 0, an algorithm for determination of estimates and their covariance matrix will be as follows:

$$\hat{x}_{i/i} = a[(\hat{x}_{i/i-1} - S_{i/i-1}c); c; i-1]\Big|_{c=0} + S_{i/i} b[(\hat{x}_{i/i-1} - S_{i/i-1}c); c; i-1]\Big|_{c=0} ; \quad (23)$$

$$S_{i/i} = -\left\{ \frac{\partial a[(\hat{x}_{i/i-1} - S_{i/i-1}c); c; i-1]}{\partial c}\Bigg|_{c=0} \right\} \times \left\{ \frac{\partial b[(\hat{x}_{i/i-1} - S_{i/i-1}c); c; i-1]}{\partial c}\Bigg|_{c=0} \right\}^{-1} . \quad (24)$$

Partial derivatives that enter into Eqs. (23) and (24) are determined from relations (19) and (20), taking account of expression (21) and assumptions (22), i.e.,

$$\frac{\partial a}{\partial c} = -S_{i-1/i-1} - N_{i-1}; \quad (25)$$

$$\frac{\partial b}{\partial c} = I + (\tilde{x}^T_{i-1}\Phi^T_i P^{-1}_{i/i}\Phi_i \tilde{x}_{i-1})(S_{i-i/i-1} + N_{i-1}), \quad (26)$$

where $S_{i-1/i-1}$ is an eigenvalue of the matrix of weight coefficients, which was obtained at the (i-1)-th instant of time, having regard to i -1 observations.

Upon substitution of relations (25), (26) into Eqs. (23), (24) and on performance of algebraic transformations with the use of a lemma on matrix inversion, we obtain the following algorithm for error estimation.

Prediction: $\hat{\Lambda}_{i/i-1} = \hat{\Lambda}_{i-1/i-1}$; (27)

$$S_{i/i-1} = S_{i-1/i-1} + N_i. \quad (28)$$

Updating: $S_{i/i} = S_{i/i-1} - S_{i/i-1}\hat{\tilde{x}}^T_{i/i-1}V_i^{-1}\hat{\tilde{x}}_{i/i-1}S_{i/i-1}$; (29)

$$V_i = (\hat{\tilde{x}}_{i/i-1}S_{i/i-1}\hat{\tilde{x}}^T_{i/i-1} + P_{i/i}); \quad (30)$$

$$\hat{\Lambda}_{i/i} = \hat{\Lambda}_{i/i-1} + S_{i/i}\hat{\tilde{x}}^T_{i/i-1}P^{-1}_{i/i}\eta_i, \quad (31)$$

where $\hat{\tilde{x}}_{i/i-1} = diag\{\hat{x}_{1(i/i-1)},...,\hat{x}_{n(i/i-1)}\}$; $\hat{\Lambda}_{i/i-1}$; $\hat{\Lambda}_{i-1/i-1}$ are respectively the predicted and updated (by the residual η_i) estimates of the vector of coefficients that characterize the values of uncompensated errors; $S_{i/i-1}$; $S_{i/i}$ are respectively the predicted and updated covariance matrices for the errors of forming the compensation coefficients, which are caused by dynamic noise environment.

In the execution of the algorithm (27) – (31), the need arises for inversion of the matrix V_i in Eq. (29). Such a problem can be solved when the *U-D* technology is employed for the realization of the EKF, see Bierman (1977). In this case,

$$M_j = M_{j-1} - M_{j-1}\tilde{U}^T_j\tilde{U}_j M_{j-1}/D_{jj}; \; j = \overline{1,n}; \quad (32)$$

$$M_0 = S_{i/i-1}; \; S_{i/i} = M_n,$$

where $\tilde{U}_j$ is the j-th row of the matrix $U^{-1}_{i/i}\hat{\tilde{x}}_{i/i-1}$;

D_{jj} is the j-th element of the diagonal matrix $D_{i/i}$;

$U_{i/i}D_{i/i}U^T_{i/i} = P_{i/i}$ is the covariance matrix of estimation errors, which is formed by the EKF at the i-th instant of time from i observations; $U_{i/i}$ is the upper triangular matrix with identity diagonal elements; n is the dimension of the vector of DS errors x_i.

In order that relation (32) be realized, it is necessary to include the following procedures in the *U-D* filter algorithm as an addition to it.

Prediction: $m_0 = \hat{x}_{i/i-1} = \tilde{\Phi}_i\hat{x}_{i-1/i-1}$;

$$MWGS \left\{ \begin{array}{l} \overline{W}_i = [\tilde{\Phi}_i U_{i-1/i-1} \vdots \Gamma_i] \\ \overline{D}_i = diag(D_{i-1/i-1}, Q_{i-1}) \end{array} \right\} \rightarrow U_0; D_0;$$

$$f_j = \Gamma^T_j U^{-T}_{j-1}; \; V_j = D^{-1}_{j-1}f^T_j;$$

$$K_j = U^{-T}_{j-1}V_j/(f_jV_j + Q^{-1}_{jj});$$

$$MWGSL \left\{ \begin{array}{l} \tilde{W}_j = [K_jf_j - U^{-T}_j \vdots K_j] \\ \tilde{D}_j = diag(D^{-1}_j, Q^{-1}_{jj}) \end{array} \right\} \rightarrow U^{-T}_j; D^{-1}_j; \; j = \overline{1,r}.$$

Updating: $\nu_j = z_j - H_jm_{j-1}$; $\beta_j = \nu_j/\sqrt{R_j}$;

$$f_j = H_jU_{j-1}; \; V_j = D_{j-1}f^T_j;$$

$$\tilde{\alpha}^2_j = f_jV_j\psi'_j + R_j; \; K_j = U_{j-1}V_j/\tilde{\alpha}^2_j;$$

$$m_j = m_{j-1} + K_j\sqrt{R_j}\psi_j;$$

$$MWGS \left\{ \begin{array}{l} \overline{W}_j = [K_jf_j\psi'_j - U_{j-1} \vdots K_j] \\ \overline{D}_j = diag(D_{j-1}, R_j\psi'_j) \end{array} \right\} \rightarrow U_j; D_j;$$

$$MWGSL \left\{ \begin{array}{l} \tilde{W}_j = [U^{-T}_{j-1} \vdots H^T_j] \\ \tilde{D}_j = diag(D^{-1}_{j-1}, \psi'_j R_j^{-1}) \end{array} \right\} \rightarrow U^{-T}_j; D^{-1}_j; \; j = \overline{1,l};$$

$$\hat{x}_{i/i} = m_l; U_{i/i} = U_l; D_{i/i} = D_l;$$

$$U^{-T}_{i/i} = (U^{-1}_{i/i})^T = U^{-T}_l; \; D^{-1}_{i/i} = D^{-1}_l;$$

Identification: (27) – (32),

where *MWGS* is the procedure for orthogonal transformation, see Bierman (1977), of the combination of the $n\times(n+r)$ rectangular matrix $\overline{W}_j$ and the $(n+r)\times(n+r)$ diagonal matrix $\overline{D}_j$ into the combination of the upper triangular matrix U_j with unit diagonal elements and the $n\times n$ diagonal matrix D_j; *MWGSL* is a procedure that is similar to the *MWGS* one, which is intended to form the lower triangular matrix U^{-T}_j with unit diagonal elements and the diagonal matrix D^{-1}_j, see Chernodarov (2017); H_j is the row vector of coefficients of coupling between the observation z_j and the vector of errors m_j; l is the dimension of the vector $Z_i = Z(t_i) = [z_{1(i)}...z_{j(i)}...z_{l(i)}]$ of observations; Γ_j is the j-th column of the matrix Γ_i, which is a matrix of dimension $n\times r$; Γ_i is the transition matrix for the disturbance vector ξ_i; Q_{jj} is the j-th element of the diagonal matrix Q_i, which is a matrix of dimension $r\times r$; $\psi_j = \psi(\beta_j) = \left.\frac{\partial\rho(\beta)}{\partial\beta}\right|_{\beta=\beta_j}$;

$\psi'_j = \psi'(\beta_j) = \left.\frac{\partial^2\rho(\beta)}{\partial\beta^2}\right|_{\beta=\beta_j}$; H_j is the row vector of coefficients of coupling between the observation Z_j and the vector of errors m_j.

The influence function $\psi(\beta)$, see Wu (1993), determines the level of confidence in incoming observed values. The functions ψ_j and ψ'_j can be formed with due regard for «a priori» assumptions as to the distribution laws of the valid signal and noise. The following values of the function $\psi(\beta)$, see Chernodarov (2009), have been suggested and justified, which take account of "a priori" assumptions as to the distribution laws of the useful signal and noise:

$\psi_g(\beta_j) = \beta_j$; $\psi'_g(\beta_j) = 1$ for conditioned values of residuals, i.e., for $0 \le \beta_j \le 3$;

$\psi_l(\beta_j) = |1|$; $\psi'_l(\beta_j) = 0$ for outlying values of residuals, i.e., for $\beta_j > 6$;

$\psi_{\text{lg}}(\beta_j) = \beta/3$; $\psi'_{\text{lg}}(\beta_j) = 1/3$ for the values of residuals under the conditions of distribution uncertainty, i.e., for $3 < \beta_j \le 6$.

The algorithm (27) – (32), which was presented here reflects the technology intended for adaptive damping of DS errors from their estimates that are formed by the RKF.

3. ANALYSIS OF THE RESULTS OF STUDIES

The identification technology considered above has been approved in a natural experiment. As an object of experimentation, the SINS-500 strapdown inertial satellite navigation system, see Chernodarov et al. (2010), based on fiber-optic gyros (FOGs) has been used. In the SINS-500 system, relative-velocity components and also attitude parameters and navigation ones are reckoned through the solution of nonlinear differential equations of the 11-th degree and of the form (2), namely:

$$\dot{\bar{V}} = C_2^T\, \bar{a} + \bar{g} - 2\bar{\Omega} \times \bar{V} - \bar{\omega} \times \bar{V} - \bar{\Omega} \times \left(\bar{\Omega} \times \bar{R}\right);$$

$$2\dot{\bar{q}}_0 = \Pi \bar{q}_0;\ 2\dot{\bar{q}}_1 = \Pi \bar{q}_1, \text{ where}$$

$$\Pi_0 = \begin{bmatrix} 0 & -\dot{\Theta}_x & -\dot{\Theta}_y & -\dot{\Theta}_z \\ \dot{\Theta}_x & 0 & \dot{\Theta}_z & -\dot{\Theta}_y \\ \dot{\Theta}_y & -\dot{\Theta}_z & 0 & \dot{\Theta}_x \\ \dot{\Theta}_z & \dot{\Theta}_y & -\dot{\Theta}_x & 0 \end{bmatrix};$$

$$\Pi_1 = \begin{bmatrix} 0 & -\omega_\xi & -\omega_\eta & -\omega_\zeta \\ \omega_\xi & 0 & \omega_\zeta & -\omega_\eta \\ \omega_\eta & -\omega_\zeta & 0 & \omega_\xi \\ \omega_\zeta & \omega_\eta & -\omega_\xi & 0 \end{bmatrix};$$

$\bar{V} = [V_\xi V_\eta V_\zeta]^{\text{T}}$ is the relative-velocity vector of SINS motion, given by its components along the axes of the semiwander azimuth reference frame $o\xi\eta\zeta$, see Babich (1991); $\bar{\Omega} = [\Omega_\xi \Omega_\eta \Omega_\zeta]^{\text{T}}$ is the vector of the angular velocity of Earth rotation, given by its components along the axes of the reference frame; $\dot{\bar{\Theta}} = [\dot{\Theta}_x \dot{\Theta}_y \dot{\Theta}_z]^{\text{T}}$ is the vector of FOG output signals, given by its components along the axes of the inertial measurement unit (IMU); $\bar{\omega} = [\omega_\xi \omega_\eta \omega_\zeta]^{\text{T}}$ is the vector of turn rates of the reference frame $o\xi\eta\zeta$ in the geodetic frame; $\bar{a} = [a_x a_y a_z]^{\text{T}}$ is the vector of output signals of accelerometers; $\bar{g} = [g_\xi g_\eta g_\zeta]^{\text{T}}$ is the vector of gravitational acceleration; R is the radius vector of SINS position; $\bar{q}_0$ is a quaternion that characterizes the angular position of the frame *oxyz,* which is fixed to the IMU, with respect to the inertial frame $OX_IY_IZ_I$; $\bar{q}_1$ is a quaternion that characterizes the angular position of the wander azimuth reference navigation frame $o\xi\eta\zeta$ with respect to the ECEF frame $OX_EY_EZ_E$, see Babich (1991). From the elements of these quaternions one can find the angles ψ, ϑ, γ of IMU angular position with respect to the local geodetic frame *oENH*, along with the geodetic latitude φ and geodetic longitude λ of the SINS position.

The vector of errors comprises 18 parameters, namely: errors in the reckoning of components of the vector of relative velocity; errors in the reckoning of elements of the navigation and attitude quaternions; FOG angular drifts; accelerometer biases and an error in the reckoning of altitude relative to the Earth ellipsoid. The vector of SINS errors is estimated with the EKF by processing the following observations.

In the initial-alignment mode:

$$Z_{\Theta(i)} = C_{0(i)}^{\text{T}} \int_{t_{i-1}}^{t_i} \dot{\bar{\Theta}}(\tau) d\tau - [0 \vdots 0 \vdots \Omega \Delta t_i]^{\text{T}};$$

$$Z_{k(i)} = [\varphi_i \lambda_i]_{\text{SINS}}^{\text{T}} - [\varphi_i \lambda_i]_{\text{PIA}}^{\text{T}};$$

$$Z_{V(i)} = [V_\xi V_\eta V_\zeta]_{(i)\text{SINS}}^{\text{T}}.$$

In the navigational mode:

$$Z_{k(i)} = [\varphi_i \lambda_i]_{\text{SINS}}^{\text{T}} - [\varphi_i \lambda_i]_{\text{GPS}}^{\text{T}};$$

$$Z_{V(i)} = C_{3(i)}^{\text{T}} [V_\xi V_\eta V_\zeta]_{(i)\text{SINS}}^{\text{T}} - [V_E V_N V_H]_{(i)\text{GPS}}^{\text{T}},$$

where PIA stands for the position of the initial alignment; $\Delta t_i = t_i - t_{i-1}$ is an observation step; C_0 is the direction cosine matrix that characterizes the angular position of the SINS-fixed frame with respect to the inertial frame; C_3 is the direction cosine matrix that characterizes the angular position of the reference frame $o\xi\eta\zeta$ with respect to the geodetic frame *oENH.*

The integration step for SINS equations was 1 ms, the frequency of forming the observations was 1 Hz. The results of a comparative analysis of the operation of the SINS-500 system, see Chernodarov et al. (2010), when using different schemes for the damping of error estimates were obtained, by the reckoning of motion parameters, from the recorded signals both of sensors and of the Global Positioning System

(GPS). Experiments have been carried out under terrestrial conditions when the necessary equipment was housed in a mobile laboratory on the basis of an automobile. Figure 2 shows the horizontal path when the testing laboratory is moving under urban conditions, where

$$\Delta\varphi_{\mathrm{R}} = [\varphi_{\mathrm{GPS}}(t) - \varphi_{\mathrm{GPS}}(t_0)]R\,;\ \Delta\lambda_R = [\lambda(t) - \lambda(t_0)]R\cos\varphi$$

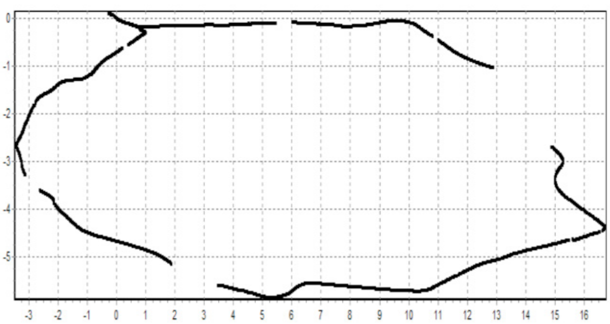

Fig.2. Horizontal path of the testing-laboratory motion under urban conditions

Certain of the results of the experiments are shown in the following drawings: the circular position error ΔS when realizing the loop for the updating of parameters (9) is depicted in Fig. 3; in Fig. 4 is shown the error ΔS when the loop for adaptive updating with identification of the coefficients of estimate damping (27) − (32) is realized, where

$$\Delta S = \sqrt{\delta_\varphi^2 + \delta_\lambda^2}\,;\ \delta_\varphi = (\varphi_{\mathrm{SINS}} - \varphi_{\mathrm{GPS}})R\,;$$

$$\delta_\lambda = (\lambda_{\mathrm{SINS}} - \lambda_{\mathrm{GPS}})R\cos\varphi_{\mathrm{GPS}}\,;$$

$$R = a\,(1 - 0.5e^2\sin^2\varphi)\,;\ a = 6378245\text{m};\ e^2 = 0.0066934\,.$$

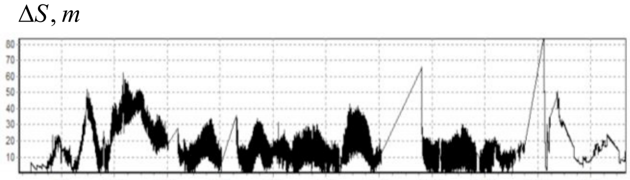

Fig. 3. Circular position error when the loop for the updating of SINS parameters (9) is realized

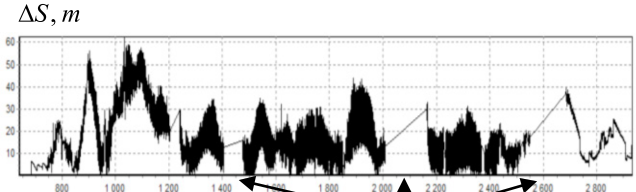

Fig. 4. Circular position error when the loop for adaptive updating of SINS parameters is realized

A comparison of the results shows that estimates obtained by the EKF with the updating loop (9) may prove to be divergent. Under the same conditions, the use of adaptive identification procedures (27) − (32) permits one to raise the consistency of estimation of SINS errors.

4. CONCLUSIONS

In the present paper, the problem of adaptive updating of the parameters of nonlinear observable DSs from the estimates of their errors has been formulated. The proposed solution of the above problem is based on the application of the method of inverse problems of dynamics. The implementation of such a method relies on the identification of damping coefficients for the estimates of DS errors between observation sessions. The identification is performed on the basis of processing the residuals between the predicted estimates and the estimates updated from observations. The studies conducted and the results obtained corroborate the effectiveness of employing the procedures of inverse filtering in identification problems.

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